

Alpha Individual Test
 Mu Alpha Theta State Convention 2000

The abbreviation NOTA denotes
 "None of These Answers"

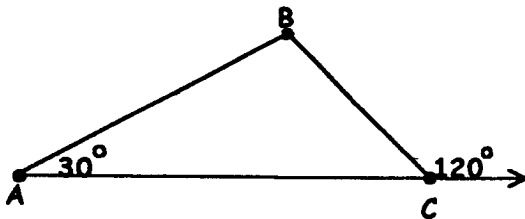
1. If $\sin \theta = \frac{1}{3}$ for $0 < \theta < \frac{\pi}{2}$ then give the value of $\cos \theta$.

- A. $\frac{\sqrt{2}}{4}$ B. $\frac{8}{9}$
 C. $\frac{3}{4}$ D. $\frac{2\sqrt{2}}{3}$ E. NOTA

2. For $i = \sqrt{-1}$ expand and simplify $(1 + i)^4$.

- A. 2 B. -2
 C. 4 D. -4 E. NOTA

3.



If $AC=10$ then find the value of BC .

- A. 5 B. $5\sqrt{2}$
 C. 6 D. $5\sqrt{3}$ E. NOTA

4. If $\frac{1}{\sec^2 x} + \frac{\cos x}{\tan x} = f(x) \cdot (1 + \csc x)$
 then for $0 < x < \frac{\pi}{2}$, $f(x) =$

- A. $\cos^2 x$ B. $\sec^2 x$
 C. $\tan^2 x$ D. $\csc^2 x$
 E. NOTA

5. Which of the following statements is true?

- A. A triangle with sides of 3 cm, 4 cm and 5 cm has angles of measures 30, 60 and 90 degrees.
 B. A triangle with sides of 3 cm, 4 cm and 5 cm is similar to one that has angles of 30, 60 and 90 degrees.
 C. A triangle with angles of 30, 60 and 90 degrees may have sides of 3 cm, 4 cm and 5 cm.
 D. A triangle with sides of 3 cm, 4 cm and 5 cm has a perimeter in cm that is equal to its area in sq. cm.
 E. NOTA

6. For how many values of x is the equation $\sin x = \cos x$ true, for $0 < x < 6$ radians?

- A. 0 B. 1
 C. 2 D. 3 E. NOTA

7. If $f(x) = \frac{1}{x}$ then which is an equivalent expression for $f(f(f(\csc 2^\circ)))$?

- A. $\sin 2^\circ$ B. $\csc 2^\circ$
 C. $\sin^2 2^\circ$ D. 1 E. NOTA

8. The function $f(x) = x^2 - 5x + c$ has a root at $x = -3$. What is the value of the other root?

- A. 2 B. -2
 C. 8 D. 15 E. NOTA

9. If $\cos \theta + \cos^2 \theta + \cos^3 \theta + \dots = \frac{3}{4}$
then $\cos \theta =$

- A. $\frac{2}{3}$ B. $\frac{3}{7}$
C. $\frac{1}{4}$ D. $\frac{3\sqrt{2}}{4}$ E. NOTA

10. Simplify: $\sqrt{6 - \sqrt{6 - \sqrt{6 - \sqrt{6 - \dots}}}}$

- A. 1 B. 2
C. 3 D. 4 E. NOTA

11. A triangle has sides of lengths 3 cm, 5 cm and 6 cm. Give the length of the longest altitude in the triangle.

- A. $\frac{4\sqrt{14}}{3}$ B. $\frac{\sqrt{14}}{3}$
C. 6 D. 5 E. NOTA

12. What is the sum of the focal radii of the ellipse given by the equation $4x^2 + 9y^2 = 36$?

- A. 2 B. 3
C. 4 D. 9 E. NOTA

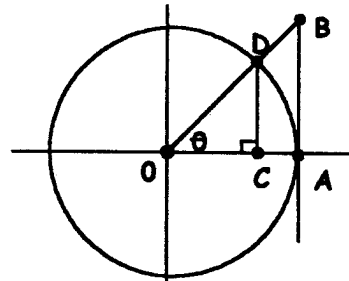
13. Simplify: $\frac{\sqrt{2a} - \sqrt{a}}{\sqrt{2a} + \sqrt{a}}$ for $a > 0$

- A. $3 - 2\sqrt{2}$ B. $\frac{299 - 198\sqrt{2}}{100}$
C. $\frac{\sqrt{29583}}{100}$ D. $2 + 3\sqrt{2}$
E. NOTA

14. The equation $2y^2 - 4x^2 = 8$ defines a hyperbola with asymptotes' equations $y = mx$ and $y = -mx$ for $m > 0$. Give the value of m .

- A. 2 B. 4
C. $\sqrt{2}$ D. $\frac{\sqrt{2}}{2}$ E. NOTA

15.



\overline{AB} is tangent to circle O and $OC = \cos \theta$, $CD = \sin \theta$, $OD = 1$. Which is an expression for the length AB ?

- A. $\sec \theta$ B. $\tan \theta$
C. $\csc \theta$ D. $\cot \theta$ E. NOTA

16. If $\begin{vmatrix} \cos \theta & 4 & -1 \\ \sin \theta & 1 & 0 \\ 0 & 3 & 2 \end{vmatrix} = 0$ and

$\cos \theta = k \sin \theta$, then $k = \dots$

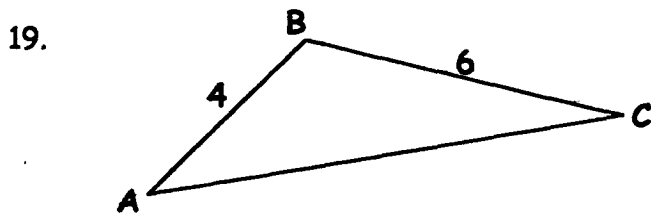
- A. $\frac{2}{11}$ B. $\frac{2}{5}$
C. $\frac{11}{2}$ D. $\frac{5}{2}$ E. NOTA

17. For $2^{2x} = \left(\frac{1}{8}\right)^k$ then solve for x in terms of k .

- A. $\frac{3}{2}k$ B. $\frac{2}{3}k$
C. $-\frac{3}{2}k$ D. $-\frac{2}{3}k$ E. NOTA

18. The greatest power of 10 that is a factor of $3^8 \cdot 8^6 \cdot 5^{20}$ is 10^k . Find the value of k.

- A. 23 B. 20
C. 19 D. 18 E. NOTA



In triangle ABC shown, B is an obtuse angle. If k is the greatest lower bound for length AC, and x is the least upper bound for AC, $k < AC < x$ then, $x^2 + k^2 =$

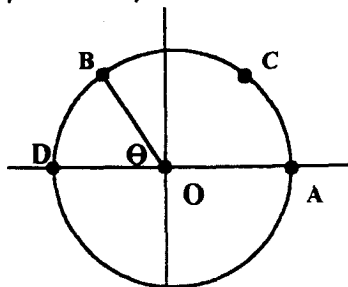
- A. 52 B. 72
C. 104 D. 152 E. NOTA

20. The sequence 0, 1, 4, 5 is repeated twenty more times, to give 84 numbers to the sequence. What is the sum of the 84 numbers?

- A. 400 B. 210
C. 200 D. 180 E. NOTA

21. In circle O, radius \overline{OB} has length 12 and arc ACB has length 8π . What is the measure of angle θ , $\angle BOD$, in radians?

- A. $\frac{\pi}{3}$ B. $\frac{2\pi}{3}$
C. $\frac{\pi}{6}$ D. $\frac{5\pi}{6}$



may not be drawn to scale

E. NOTA

22. In triangle ABC, $AB=12$, $BC=10$ and $m\angle A = 32^\circ$ (degrees). Which is a possible value for the length of side \overline{AC} , to the nearest hundredths place?

- A. 8.29 B. 5.18
C. 11.84 D. 17.89 E. NOTA

23. If $f(x) = 4x^2 - 3x + 1$ and $g(x) = 9$ then find the value of $g(f(1))$.

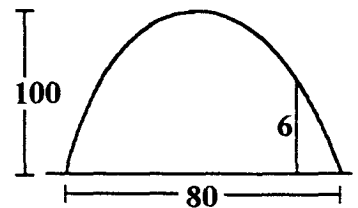
- A. 2 B. 9
C. 11 D. 18
E. NOTA

24. There are 5 baseball teams and they each play a warm-up game with each other team. All warm-up games together take 100 minutes and no games are played simultaneously. How many minutes will each team play in the entire "warm-up?"

- A. 10 B. 20
C. 30 D. 40 E. NOTA

25. A parabolic arch is 100 feet tall at its apex and the inner surfaces of its feet are 80 feet apart. If a six-foot tall person can walk beneath the arch and have his head just touch the surface of the arch, how far is he from the center (axis of symmetry) of the arch? Round to the nearest hundredth place.

- A. 34.56 feet
B. 37.75 feet
C. 38.78 feet
D. 77.56 feet
E. NOTA



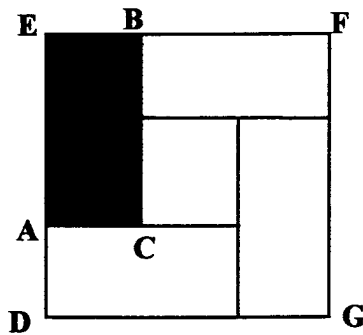
26. The functions $f(x) = x^3 + 3x^2 - 4x - 12$ and $g(x) = x^2 - 2x - 15$ share a root, k . Give the value of $f(k) + g(k) + k$.

A. 3 B. 5
C. -3 D. -5 E. NOTA

27. The square EFGD consists of four congruent rectangles and a smaller square, none of which have overlapping areas. One of the four congruent rectangles, shaded, has dimensions $\cos \theta$ and $\sin \theta$, for $0 < \theta < \frac{\pi}{2}$.

If the area of the square EFGD is $\frac{9}{8}$ then find the value of $\sin(2\theta)$.

A. $\frac{1}{4}$
B. $\frac{1}{8}$
C. $\frac{1}{16}$
D. $\frac{8}{15}$
E. NOTA



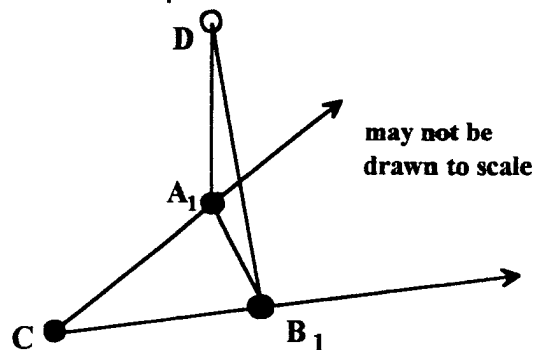
28. A stock's price increases and decreases according to the function $S(x) = 2 \cos\left(\frac{x}{\pi} + 1\right) + 4$, where x is the number of days after January 1, 2000 and S is the increase in dollars per share from the previous day's price. Which of the following statements must be true of the stock's price in the week of January 2 through January 8, 2000?

A. The stock's price never decreases.
B. The stock's minimum price is \$2.
C. The stock's maximum price is \$4.
D. The stock's average price is approximately \$4.
E. NOTA

29. Juliet has an account which has paid 4% interest, compounded annually. When Juliet's money had been in the bank for two years, Simone put in an account an amount identical to Juliet's original amount. (Same bank, same interest rate.) After ten years from Simone's initial deposit, Simone had \$12,000. Neither woman withdrew or deposited money from the accounts, except for the original deposits. How much money did Juliet have at that same time, rounded to the nearest dollar?

A. \$12979 B. \$11095
C. \$15000 D. \$9600 E. NOTA

30. Boat A launches from point C at noon, with a bearing of 45 degrees (clockwise off north). At 12:30 p.m. boat B launches from the same point with a bearing of 80 degrees. Boat A travels a linear path at 4 miles per hour (mph) and boat B travels a linear path at 6 mph. At 1:00 p.m. boat A is 10 miles south of buoy D (point D) and is at position A_1 ; at that time boat B is at position B_1 . Give the area of $\triangle A_1 B_1 D$ formed at 1:00 p.m., rounded to the nearest tenth of a square mile.



A. 0.6 B. 10.4
C. 0.8 D. 12.3
E. NOTA