

Alpha Applications: 2000 FAMAT Convention

1. Mary's speed on the interstate is being clocked from a helicopter in the air. The copilot can see two lines 100 yards apart. If the speed limit is 70 miles per hour, how little time can Mary spend between the lines and still avoid a speeding ticket? (rounded to the nearest hundredth of a second, please)

- a) 5.84 b) 1.46 c) 3.11 d) 2.92 e) NOTA

2. Assume ψ to be a third quadrant angle and λ to be a first quadrant angle. Find $\psi + \lambda$, please.

$$\tan(\psi + \lambda) = \frac{\tan(\psi) + 1}{1 - \frac{\sqrt{3}}{3} \cdot \tan(\lambda)}$$

- a) $\frac{7\pi}{6}$ b) $\frac{17\pi}{12}$ c) $\frac{5\pi}{12}$ d) $\frac{29\pi}{12}$ e) NOTA

3. The eccentricity of Earth's orbit around the sun is closest to which of the following values?

- a) 1 b) 10 c) -0.5 d) 0 e) NOTA

4. How many dots would be needed to draw the 56th triangular number? For you history buffs, it was also the year of Descartes' birth.

- a) 1596 b) 1653 c) 1617 d) 1408 e) NOTA

5. If a force is applied by the vector with equation $F = 3i + 4j + 5k$ to move a particle from the point (4,2,0) to the point (0,2,3), what is the work done?

- a) 43 b) -15 c) 3 d) 52 e) NOTA

6. Several MAΘ students were surveyed to see what the most popular pro football team in Florida was. The results showed that 45 people liked the Bucs, 30 people liked the Jaguars, 25 people liked the Dolphins, 14 people liked both the Bucs and the Jaguars, 16 people liked both the Jaguars and the Dolphins, 10 liked both the Bucs and the Dolphins, 8 liked all three and 31 liked none of them. How many people were subjected to this survey?

- a) 70 b) 97 c) 101 d) 99 e) NOTA

7. Match each of the following with their simplified equivalent.

I. $\frac{1 + \sin^2(\Theta) \cdot \tan^2(\Theta) - \cos^2(\Theta)}{\sin(\Theta)}$

A. $|\sin(2 \cdot \Theta)|$

II. $2\sqrt{-\cos(2 \cdot \Theta) + \cos^2(\Theta) - \sin^4(\Theta)}$

B. $\sec(\Theta) \cdot \tan(\Theta)$

III. $\frac{\sec(\Theta)}{\tan(\Theta)} \cdot \frac{\cot(\Theta)}{\cos(\Theta)} \cdot \frac{\sin(\Theta)}{\csc(\Theta)}$

C. 1

- a) I-C, II-B, III-A b) I-B, II-A, III-C c) I-B, II-C, III-A
d) I-C, II-A, III-B e) NOTA

8. When Jeff tosses a frisbee, it's horizontal velocity is 32 feet per second. Its vertical position in feet is given by the equation $h = -2t^2 + 15t + 5$. How far does the disc travel before it lands again? (to the nearest yard, please)

- a) 83 b) 250 c) 88 d) 256 e) NOTA

9. Evaluate: $\sum_{n=0}^{2000} \sqrt{\cos\left(\frac{n \cdot \pi}{2}\right)}$, please

- a) 500+501i b) 501-500i c) -501+500i d) 500-501i e) NOTA

10. A ship leaves port traveling 48° east of due north and at a rate of 20 mph. Two hours later another ship leaves the same port traveling 30° south of due east at a rate of 24 mph. How far apart are the ships when the second ship has been at sea for 3 hours? (round to the nearest tenth of a mile, please)

- a) 103.6 b) 104.9 c) 101.5 d) 98.9 e) NOTA

11. If there are 2.62×10^{22} atoms in 1.00 g of sodium and they are lined up side by side, what is the length of the line of sodium atoms in miles? Assume that the atoms are spheres of radius 1.86×10^{-10} m. Potentially useful info: 1 mile = 1.609 km.

- a) 3.04×10^{11} b) 4.87×10^{11} c) 6.09×10^{11} d) 6.06×10^9 e) NOTA

12. Given $\triangle LHS$ with $LS=7$, $LH=4$, and $HS=2\sqrt{10}$, find the length of the altitude to side LS .

- a) $\frac{14\sqrt{10}}{10}$ b) $\frac{9\sqrt{31}}{14}$ c) $\frac{31\sqrt{14}}{10}$ d) $\frac{10\sqrt{31}}{14}$ e) NOTA

13. Joe, Ben, Devlyn, and Eric take turns attempting to solve a particularly tough problem. Joe goes first and they continue in the given order. If none of them are able to figure it out, they simply start over again. If the probability for each getting the problem right is 0.5, 0.8, 0.75, and 0.4, respectively, what is the probability that Eric will correctly answer the problem first?

- a) 0 b) 0.01 c) 0.015 d) 0.0015 e) NOTA

14. For the final exam in his class, a professor gives his students 8 possible essay questions from which he will pick two to be on the exam. Each student then chooses to write one of the two. Let n be the number of questions studied by a student. If $p(n)$ represents the probability of seeing at least one question studied for on the exam and the efficiency function $E(n) = \frac{p(n)}{n}$, what $n \geq 1$ maximizes $E(n)$?

- a) 1 b) 3 c) 5 d) 7 e) NOTA

15. For what temperature does degrees Fahrenheit equal degrees Celsius?

$$F^{\circ} = \frac{9 \cdot C^{\circ}}{5} + 32$$

- a) 40° b) 42° c) -40° d) 0° e) NOTA

16. The perfect banana split is defined as having 3 different flavors of ice cream and three different toppings. Whipped cream and nuts are both optional and neither counts as a topping. If a selection of 10 ice cream flavors and 15 toppings are available, how many distinguishable banana splits are possible?

- a) 7,862,400 b) 1,965,600 c) 54,600 d) 218,400 e) NOTA

17. Let $f(x) = \frac{x^4 - 12x^3 + 54x^2 - 108x + 81}{x^2 - 7x + 12}$. Find all the asymptotes to f , please.

- a) $x = 3, x = 4, y = 0$
 b) $x = 3, y = 2x - 7$
 c) $x = 4, y = x^2 - 5x + 7$
 d) $x = 3, y = \frac{x^3}{3} - \frac{5x^2}{2} + 7x + C$
 e) NOTA

18. Austin Powers is riding high in his jumbo jet with the very shagadellic Ms. Kensington when the plane suddenly begins descending with an angle of depression of 30° . How bizarre!?! If the jet begins its descent from 30,000 feet above the ground and it is traveling at 500 mph, how long does the international man of mystery have to rectify the situation? (to the nearest second, please)

- a) 82 b) 47 c) 412 d) 88 e) NOTA

19. In the study of logic, the term that describes when all scenarios must be true is known as a...

- a) truth table b) tautology c) fact d) verbum sat sapientia e) NOTA

20. A radioactive substance has a half-life of 50 years. If 100 grams are allowed to decay for 125 years, how much of the original compound will remain? (in grams, please)

- a) $\frac{20}{\ln(3)}$ b) $\frac{25\sqrt{2}}{2}$ c) $\frac{4\sqrt{5}}{5}$ d) $\frac{5\ln(2)}{4}$ e) NOTA

21. Given: $\sum_{x=1}^{\infty} \frac{1}{x^2} = \frac{\pi^2}{6}$. Determine $\sum_{x=2}^{\infty} \frac{3x^4 + 1}{x^2(x-1)^2(x+1)^2}$, please.

- a) $\frac{\pi^2}{2} - \frac{9}{4}$ b) $\frac{\pi^4}{36} + 1$ c) $\frac{\pi^2}{6} - \frac{9}{4}$ d) $\frac{\pi^2}{2} + 1$ e) NOTA

22. How many complete revolutions does a tire make over the course of a mile if its diameter is 3 feet?

- a) 280 b) 186 c) 1120 d) 736 e) NOTA

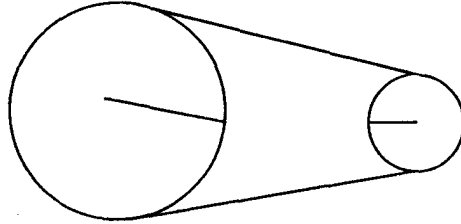
23. A gas tank is in the shape of a cylinder with two hemispheres on either end. From end to end, the tank is 4 feet long. The material used to construct the tank is a half inch thick on the inside. If the hemispheres have radii of one foot, how much gas can the tank hold? (round your answer to the nearest cubic foot, please)

- a) 8 b) 9 c) 10 d) 11 e) NOTA

24. Quadrilateral ABCD is inscribed in a circle. $\overline{AB}=1$, $\overline{BC}=3$, $\overline{CD}=4$ and \overline{BD} is a diameter of the circle. The length of $\overline{AC} = \frac{a+b\sqrt{b}}{c}$. Find a+b+c where b is not divisible by the square of any prime number, please.

- a) 23 b) 89 c) 42 d) 15 e) NOTA

25. A cord goes around the pulley system depicted below. The larger of the circles has a radius of 18 inches and the smaller has a radius of 3 inches. Their centers are 30 inches apart. A point takes 30 seconds to circumnavigate the system. What is the angular velocity of the smaller circle? (please leave your answer in units of radians per second and round it to the hundredths place)



- a) 1.20 b) 1.19 c) 1.48 d) 1.49 e) NOTA

26. On a city map Main Street is noted as having a linear equation of $y = x$. Broadway intersects Main Street at the point $(10, 10)$ and has a slope of -2 . Find the acute angle of intersection between the two roads to the nearest degree, please.

- a) 108° b) 64° c) 71° d) 63° e) NOTA

27. Evaluate $\lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin(x)}{h}$, please.

- a) $\sin(x)$ b) π c) 1 d) $\cos(x)$ e) NOTA

28. A comet enters our solar system traveling on a path in the shape of a parabola with the sun as the focus and at the origin. The equation of the directrix is found to be $y = -2.0 \times 10^8$ (miles). Find the equation of the parabola in $y = Ax^2 + Bx + C$ form, leaving each coefficient in units of 100,000,000 miles, please.

a) $y = 0x^2 + \frac{1}{2}x - 1$ b) $y = \frac{1}{4}x^2 + 0x - 1$ c) $y = \frac{1}{2}x^2 + 0x - 1$

d) $y = \frac{1}{4}x^2 - \frac{1}{2}x + 1$ e) NOTA

29) Find the polar representation of the hyperbola $x^2 - y^2 = 1$, please.

a) $r = \sqrt{-\sec(2 \cdot \Theta)}$ b) $r = \pm \sqrt{\sec(2 \cdot \Theta)}$ c) $r = \pm \sqrt{-\sec(2 \cdot \Theta)}$

d) $\sqrt{\sec(2 \cdot \Theta)}$ e) NOTA

30. Ryan picks up a MA Θ test and quickly runs through the first 29 problems without any errors. What is the probability of him making a perfect score?

a) $\frac{1}{5}$

b) $\frac{1}{6}$

c) $\frac{1}{4}$

d) $\frac{29}{30}$

e) NOTA

DON'T PANIC!



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